# A determination of the CPT violation parameter $\operatorname{Re}(\delta)$ from the semileptonic decay of strangeness-tagged neutral kaons 

## CPLEAR Collaboration

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#### Abstract

We have improved by two orders of magnitude the limit currently available for the CPT violation parameter $\operatorname{Re}(\delta)$. To this purpose we have analyzed the full sample of neutral-kaon decays to e $\pi \nu$ recorded in the CPLEAR experiment, where the strangeness of the neutral kaons was tagged at production and decay time. An appropriate function of the measured decay rates, including information from the analysis of $\pi^{+} \pi^{-}$decay channel, gives directly $\operatorname{Re}(\delta)$. The result $\operatorname{Re}(\delta)=(3.0$ $\left.\pm 3.3_{\text {stat }} \pm 0.6_{\text {syst }}\right) \times 10^{-4}$ is compatible with zero. Values for the parameters $\operatorname{Im}(\delta), \operatorname{Re}\left(x_{-}\right)$and $\operatorname{Im}\left(x_{+}\right)$were also obtained. © 1998 Elsevier Science B.V. All rights reserved.


## 1. Introduction

Within the framework of a local field theory, of Lorentz invariance and of the usual spin-statistics requirement, any order of the triple product of the discrete symmetries $\mathrm{C}, \mathrm{P}$ and T represents an exact symmetry expressed by the CPT theorem [1]. However non-local interactions, postulated in modern Grand Unified Theories, may entail a violation of CPT at very short distances. The evolution in time of a $\mathrm{K}^{0}$ (or $\overline{\mathrm{K}}^{0}$ ) and the corresponding change in semileptonic decay rates are a good tool for the study of CPT symmetry. The lack of CPT invariance would appear as an asymmetry when comparing $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$ semileptonic rates.

In this paper we report the results of an analysis performed with $1.3 \times 10^{6}$ semileptonic decays collected in the CPLEAR experiment, to determine the real part of $\delta$, the CPT parameter in the neutral-kaon mixing. Our measurement of $\operatorname{Re}(\delta)$, with an accuracy of a few $10^{-4}$, is two orders of magnitude more precise than the current value. This level of accuracy is of great importance in the analysis of CPT invariance in the neutral-kaon system, since it allows cancellations between possible CPT violations in the kaon mixing and in the decay amplitude for $\pi^{+} \pi^{-}$decays to be ruled out [2].

This measurement takes advantage of the strangeness tagging facilities of the CPLEAR experiment. There $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$ states were produced in $\mathrm{p} \overline{\mathrm{p}}$ annihilations via the reactions

$$
\begin{align*}
& \mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{~K}^{-} \pi^{+} \mathrm{K}^{0}  \tag{1}\\
& \mathrm{~K}^{+} \pi^{-} \overline{\mathrm{K}}^{0}
\end{align*}
$$

each with a branching ratio of $\approx 2 \times 10^{-3}$, enabling the neutral-kaon strangeness at the production to be tagged by the charge of the accompanying charged kaon. The strangeness of the kaon at the decay time $t=\tau$ was tagged by the lepton charge, a positive (negative) lepton charge being associated to a positive (negative) strangeness of the kaon.

## 2. Phenomenology

In the semileptonic decays of the neutral kaons there are four independent decay rates, depending on the strangeness of the kaon $\left(\mathrm{K}^{0}\right.$ or $\left.\overline{\mathrm{K}}^{0}\right)$ at the production time, $t=0$, and on the charge of the decay lepton ( $\mathrm{e}^{+}$or $\mathrm{e}^{-}$),

$$
\begin{array}{ll}
R_{+}(\tau) \equiv R\left[\mathrm{~K}_{t=0}^{0} \rightarrow \mathrm{e}^{+} \pi^{-} v_{t=\tau}\right], & \bar{R}_{-}(\tau) \equiv R\left[\overline{\mathrm{~K}}_{t=0}^{0} \rightarrow \mathrm{e}^{-} \pi^{+} \bar{v}_{t=\tau}\right], \\
R_{-}(\tau) \equiv R\left[\mathrm{~K}_{t=0}^{0} \rightarrow \mathrm{e}^{-} \pi^{+} \bar{v}_{t=\tau}\right], & \bar{R}_{+}(\tau) \equiv R\left[\overline{\mathrm{~K}}_{t=0}^{0} \rightarrow \mathrm{e}^{+} \pi^{-} v_{t=\tau}\right] .
\end{array}
$$

The above four rates can be parametrized as a function of the mixing parameters $\epsilon$ (T-violation parameter) and $\delta$ (CPT-violation parameter):

$$
\epsilon=\frac{\Lambda_{\overline{\mathrm{K}}^{0}, \mathrm{~K}^{0}}-\Lambda_{\mathrm{K}^{0}} \overline{\mathrm{~K}}^{0}}{2\left(\lambda_{\mathrm{L}}-\lambda_{\mathrm{S}}\right)} \quad \text { and } \quad \delta=\frac{\Lambda_{\overline{\mathrm{K}}^{0} \overline{\mathrm{~K}}^{0}}-\Lambda_{\mathrm{K}^{0}, \mathrm{~K}^{0}}}{2\left(\lambda_{\mathrm{L}}-\lambda_{\mathrm{S}}\right)} .
$$

Here, $\Lambda_{i j}$ are the elements of the effective Hamiltonian $\Lambda$ and $\lambda_{\mathrm{L}, \mathrm{S}}=m_{\mathrm{L}, \mathrm{S}}-\frac{i}{2} \Gamma_{\mathrm{L}, \mathrm{S}}$ its eigenvalues; $m_{\mathrm{L}, \mathrm{S}}$ and $\Gamma_{\mathrm{L}, \mathrm{S}}$ are the masses and decay widths for the $\mathrm{K}_{\mathrm{L}}$ and $\mathrm{K}_{\mathrm{S}}$ states, $\Delta m=m_{\mathrm{L}}-m_{\mathrm{S}}$ and $\Delta \Gamma=\Gamma_{\mathrm{S}}-\Gamma_{\mathrm{L}}$. The $\mathrm{K}_{\mathrm{L}}$ mixing parameter is defined as $\epsilon_{\mathrm{L}}=\epsilon-\delta$.

The decay amplitudes corresponding to the four rates can be written as $[2,3]$

$$
\begin{aligned}
& \left\langle\mathrm{e}^{+} \pi^{-} \nu\right| \Lambda\left|\mathrm{K}^{0}\right\rangle=a+b,\left\langle\mathrm{e}^{-} \pi^{+} \bar{\nu}\right| \Lambda\left|\overline{\mathrm{K}}^{0}\right\rangle=a^{*}-b^{*}, \\
& \left\langle\mathrm{e}^{-} \pi^{+} \bar{\nu}\right| \Lambda\left|\mathrm{K}^{0}\right\rangle=c+d,\left\langle\mathrm{e}^{+} \pi^{-} \nu\right| \Lambda\left|\overline{\mathrm{K}}^{0}\right\rangle=c^{*}-d^{*} .
\end{aligned}
$$

The amplitudes $b$ and $d$ are CPT-violating, $c$ and $d$ describe possible violations of the $\Delta S=\Delta Q$ rule, and the imaginary parts are all T -violating. The quantities

$$
x=\frac{c^{*}-d^{*}}{a+b} \quad \text { and } \quad \bar{x}=\frac{c^{*}+d^{*}}{a-b}
$$

describe the violation of the $\Delta S=\Delta Q$ rule in decays into positive and negative leptons, respectively, while $y=-b / a$ describes CPT violation in semileptonic decays in the case where the $\Delta S=\Delta Q$ rule holds. The parameters $x_{+}=(x+\bar{x}) / 2$ and $x_{-}=(x-\bar{x}) / 2$ describe therefore violation of the $\Delta S=\Delta Q$ rule in CPT conserving and CPT violating amplitudes, respectively. We assume $x, \bar{x}$ and $y \ll 1$.

By considering $\operatorname{Re}(a)$ to be of the order of unity and keeping first order terms in all other quantities, the four independent decay rates can be written as:

$$
\begin{align*}
R_{+}(\tau)= & \frac{|a|^{2}}{4}\left([1+2 \operatorname{Re}(x)+4 \operatorname{Re}(\delta)-2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{S}} \tau}+[1-2 \operatorname{Re}(x)-4 \operatorname{Re}(\delta)-2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{L}} \tau}\right. \\
& \left.+\{2[1-2 \operatorname{Re}(y)] \cos (\Delta m \tau)-[8 \operatorname{Im}(\delta)+4 \operatorname{Im}(x)] \sin (\Delta m \tau)\} \mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau}\right),  \tag{2a}\\
\bar{R}_{-}(\tau)= & \frac{|a|^{2}}{4}\left([1+2 \operatorname{Re}(\bar{x})-4 \operatorname{Re}(\delta)+2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{S}} \tau}+[1-2 \operatorname{Re}(\bar{x})+4 \operatorname{Re}(\delta)+2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{L}} \tau}\right. \\
& \left.+\{2[1+2 \operatorname{Re}(y)] \cos (\Delta m \tau)+[8 \operatorname{Im}(\delta)+4 \operatorname{Im}(\bar{x})] \sin (\Delta m \tau)\} \mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau}\right),  \tag{2b}\\
R_{-}(\tau)= & \frac{|a|^{2}}{4}\left([1+2 \operatorname{Re}(\bar{x})-4 \operatorname{Re}(\epsilon)+2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{S}} \tau}+[1-2 \operatorname{Re}(\bar{x})-4 \operatorname{Re}(\epsilon)+2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{L}} \tau}\right. \\
& \left.-\{2[1-4 \operatorname{Re}(\epsilon)+2 \operatorname{Re}(y)] \cos (\Delta m \tau)+4 \operatorname{Im}(\bar{x}) \sin (\Delta m \tau)\} \mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau}\right),  \tag{2c}\\
\bar{R}_{+}(\tau)= & \frac{|a|^{2}}{4}\left([1+2 \operatorname{Re}(x)+4 \operatorname{Re}(\epsilon)-2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{S}} \tau}+[1-2 \operatorname{Re}(x)+4 \operatorname{Re}(\epsilon)-2 \operatorname{Re}(y)] \mathrm{e}^{-\Gamma_{\mathrm{L}} \tau}\right. \\
& \left.-\{2[1+4 \operatorname{Re}(\epsilon)-2 \operatorname{Re}(y)] \cos (\Delta m \tau)-4 \operatorname{Im}(x) \sin (\Delta m \tau)\} \mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau}\right), \tag{2d}
\end{align*}
$$

where $\tau$ is the decay eigentime of the neutral kaon.
To extract the CPT violation parameter $\operatorname{Re}(\delta)$ in an optimal way we build the time-dependent decay-rate asymmetry $A_{\delta}$, defined as

$$
\begin{equation*}
A_{\delta}(\tau) \equiv \frac{\bar{R}_{+}-R_{-}\left(1+4 \operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)\right)}{\bar{R}_{+}+R_{-}\left(1+4 \operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)\right)}+\frac{\bar{R}_{-}-R_{+}\left(1+4 \operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)\right)}{\bar{R}_{-}+R_{+}\left(1+4 \operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)\right)} \tag{3}
\end{equation*}
$$

Since $\operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)=\operatorname{Re}(\epsilon)-\operatorname{Re}(\delta)$, Eq. (3) can be written, to first order in the small parameters, as

$$
\begin{align*}
A_{\delta}(\tau)= & 2 \frac{\operatorname{Im}\left(x_{+}\right) \mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau} \sin (\Delta m \tau)+\operatorname{Re}\left(x_{-}\right) \mathrm{E}_{-}(\tau)}{\mathrm{E}_{+}(\tau)-\mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau} \cos (\Delta m \tau)} \\
& +\frac{-4 \operatorname{Re}(\delta) \mathrm{E}_{-}(\tau)-2 \operatorname{Re}\left(x_{-}\right) \mathrm{E}_{-}(\tau)+\left[2 \operatorname{Im}\left(x_{+}\right)+4 \operatorname{Im}(\delta)\right] \mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau} \sin (\Delta m \tau)}{\mathrm{E}_{+}(\tau)+\mathrm{e}^{-\frac{1}{2}\left(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}}\right) \tau} \cos (\Delta m \tau)} \\
& +4 \operatorname{Re}(\delta), \tag{4}
\end{align*}
$$

with

$$
\begin{equation*}
\mathrm{E}_{ \pm}(\tau)=\frac{\left(\mathrm{e}^{-\Gamma_{\mathrm{s}} \tau} \pm \mathrm{e}^{-\Gamma_{\mathrm{L}} \tau}\right)}{2} \tag{5}
\end{equation*}
$$

We note that the asymmetry $A_{\delta}(\tau)$ does not depend on the parameter $y$. For lifetimes comparable with $1 / \Gamma_{\mathrm{S}}, A_{\delta}$ is sensitive to $\operatorname{Im}(\delta), \operatorname{Im}\left(x_{+}\right)$and $\operatorname{Re}\left(x_{-}\right)$, while for long lifetimes it depends only on $\operatorname{Re}(\delta)$, becoming simply

$$
\begin{equation*}
A_{\delta}(\tau)=8 \operatorname{Re}(\delta) \tag{6}
\end{equation*}
$$

thus allowing $\operatorname{Re}(\delta)$ to be measured without any assumption on the $\Delta S=\Delta Q$ rule.

## 3. The detector

The CPLEAR detector is described elsewhere [4] and only a brief outline is presented here. It had a cylindrical geometry and was mounted inside a solenoid of length 3.6 m and internal radius 1 m , which produced a magnetic field of 0.44 T parallel to the $\overline{\mathrm{p}}$ beam. The experiment used an intense $200 \mathrm{MeV} / \mathrm{c}$ antiproton beam ( $\approx 10^{6} \overline{\mathrm{p}} / \mathrm{s}$ ) from the Low Energy Antiproton Ring (LEAR) at CERN, which stopped in the target at the centre of the detector. The target, consisting of a $7-\mathrm{cm}$ radius sphere filled with gaseous hydrogen at 16 -bar pressure, was replaced in mid 1994 by a $1.1-\mathrm{cm}$ radius, 27 -bar, cylindrical target surrounded by a $1.5-\mathrm{cm}$ radius, cylindrical, proportional chamber (PC0).

The tracking of the annihilation products was performed by two layers of proportional chambers, six layers of drift chambers and two layers of streamer tubes. A scintillator-Cherenkov-scintillator (S1-C-S2) sandwich (PID) provided input to a fast trigger system to identify the charged kaon, based on energy loss, time of flight and Cherenkov light measurements. An 18-layer, lead/gas-sampling electromagnetic calorimeter completed the detector.

Because of the small branching ratio of the desired annihilation channels, Eq. (1), and the high beam intensity, a multi-level trigger system [4], based on custom-made hardwired processors, was used to provide fast and efficient background rejection. The PC0 information was incorporated into the trigger for all data taken during 1995 and 1996. There was a requirement of not more than two hits in this chamber, thus ensuring that the neutral kaon decayed outside PC0.

## 4. Event selection

The desired $\mathrm{p} \overline{\mathrm{p}}$ annihilations followed by the decay of a neutral kaon into $\mathrm{e} \pi \nu$ are selected by demanding events with four tracks and zero total charge. A good reconstruction quality is required for each track and
vertex. The transverse momentum of the charged kaon has to be greater than $350 \mathrm{MeV} / c$. The distance between the primary and secondary vertices in the transverse plane must be greater than 1 cm , thus removing ambiguities on the track assignment to either vertex and reducing the background from other $\mathrm{p} \overline{\mathrm{p}}$ annihilation channels. The selection of the $\mathrm{e} \pi \nu$ channel is done by identifying one of the secondary tracks as an electron or a positron, using a Neural Network (NN) algorithm [5]. This algorithm uses the momentum of the particle, the energy loss in the two scintillators (S1, S2), the number of photoelectrons in the Cherenkov counter and the time of flight from the decay vertex to the first scintillator (S1). The algorithm has been optimized using pion data from decays of neutral kaons into $\pi^{+} \pi^{-}$and electron data using $\mathrm{e}^{+} \mathrm{e}^{-}$pairs recorded in special runs from $\gamma$ conversions in the detector materials. No attempt was made to identify muons with the NN. The probability to misidentify a pion as an electron is about $2 \%$ while the probability to identify a muon from semileptonic decay as an electron, and thus contribute to the signal, is about $15 \%$.

The events are then passed through the following kinematic and geometric fits:

- 1C-fit, requiring the $\mathrm{K}^{0}\left(\overline{\mathrm{~K}}^{0}\right)$ missing mass at the primary vertex to validate the hypothesis of the $\mathrm{K}^{ \pm} \pi^{\mp} \mathrm{K}^{0}\left(\overline{\mathrm{~K}}^{0}\right)$ channel. The event is kept only if the fit yields a probability greater than $10 \%$.
- 6C-fit, requiring energy-momentum conservation under the assumption of a missing neutrino and the alignment of the $\mathrm{K}^{0}$ momentum vector with the line joining the primary and secondary decay vertices. The event is kept only if the probability is greater than $5 \%$. The fitted momenta and vertices resulting from this fit determine the decay time with a precision of $0.05 \tau_{\mathrm{S}}$ in the short lifetime region and of $0.2 \tau_{\mathrm{S}}$ in the long one.
- 4C-fit, which is performed in order to reduce the $\mathrm{K}^{0}\left(\overline{\mathrm{~K}}^{0}\right) \rightarrow \pi^{+} \pi^{-}$background that is dominant at short lifetimes, by removing the events that fit the above hypothesis. We reject the event if the probability to fit the hypothesis is greater than $10 \%$.
A total of $1.3 \times 10^{6}$ semileptonic events having a measured decay time above $1 \tau_{\mathrm{S}}\left(\tau_{\mathrm{S}} \equiv \mathrm{K}_{\mathrm{S}}\right.$ mean life $)$ survive the analysis.


## 5. Construction of the asymmetry

The detection efficiencies of $\mathrm{K}^{+} \pi^{-}$and $\mathrm{K}^{-} \pi^{+}$pairs, used to tag $\overline{\mathrm{K}}^{0}$ and $\mathrm{K}^{0}$, respectively, are not identical due to the different strong interaction cross-sections of opposite-charge kaons and pions with matter. The difference of the two efficiencies is of the order of $12 \%$. To restore the initial symmetry at the production of $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$, we introduce a normalisation factor $\xi=\epsilon\left(\mathrm{K}^{+} \pi^{-}\right) / \epsilon\left(\mathrm{K}^{-} \pi^{+}\right)$which is the ratio of the detection efficiences for the $\mathrm{K}^{+} \pi^{-}$and $\mathrm{K}^{-} \pi^{+}$pairs. Similarly, the detection efficiencies of the two final states $\mathrm{e}^{-} \pi^{+} \bar{\nu}$ and $\mathrm{e}^{+} \pi^{-} v$ are not identical. To take this into account, we introduce a second normalization factor $\eta=\epsilon\left(\pi^{+} \mathrm{e}^{-}\right) / \epsilon\left(\pi^{-} \mathrm{e}^{+}\right)$.

Correcting the measured numbers of semileptonic decays with these factors, we construct the asymmetry

$$
\begin{equation*}
A_{\delta}^{\exp }(\tau) \equiv \frac{\eta \bar{N}_{+}(\tau)-\alpha_{2 \pi} N_{-}(\tau)}{\eta \bar{N}_{+}(\tau)+\alpha_{2 \pi} N_{-}(\tau)}+\frac{\bar{N}_{-}(\tau)-\alpha_{2 \pi} \eta N_{+}(\tau)}{\bar{N}_{-}(\tau)+\alpha_{2 \pi} \eta N_{+}(\tau)}, \tag{7}
\end{equation*}
$$

where $N_{+}\left(N_{-}\right)$and $\bar{N}_{+}\left(\bar{N}_{-}\right)$stand for the observed numbers of initial $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$ accompanied by a decay e ${ }^{+}$ ( $\mathrm{e}^{-}$) and the parameter $\alpha_{2 \pi}$ is defined as $\alpha_{2 \pi}=\left[1+4 \operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)\right] \xi$.

Taking advantage of the fact that $\xi$ is independent of the decay mode, the parameter $\alpha_{2 \pi}$ can be directly determined from the CPLEAR neutral-kaon decays into $\pi^{+} \pi^{-}$[6]. We select events with decay times between 1 and $4 \tau_{\mathrm{S}}$ and build the ratio of observed $\mathrm{K}^{0}$ to $\overline{\mathrm{K}}^{0}$ events. This ratio must follow the neutral-kaon time evolution, and, in the time interval considered, can be approximated as

$$
\begin{equation*}
\left[1+4 \operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)\right] \xi \frac{N\left(\mathrm{~K}_{t=0}^{0} \rightarrow \pi^{+} \pi_{t=\tau}^{-}\right)}{N\left(\overline{\mathrm{~K}}_{t=0}^{0} \rightarrow \pi^{+} \pi_{t=\tau}^{-}\right)}=1+4\left|\eta_{+-}\right| \cos \left(\Delta m \tau-\phi_{+-}\right) \mathrm{e}^{\frac{1}{2} \Gamma_{\mathrm{s}} \tau} \tag{8}
\end{equation*}
$$

The oscillating factor on the right-hand side of Eq. (8) depends on the neutral-kaon parameters $\eta_{+-}, \Delta m$ and $\Gamma_{\mathrm{S}}$, for which we take the world averages of Ref. [7], and, as a result, is known with a precision of $\approx 1 \times 10^{-4}$. We stress that the analysis of the $\pi^{+} \pi^{-}$decay channel gives exactly the quantity $\alpha_{2 \pi}=\xi\left(1+4 \operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)\right)$ which enters the asymmetry presented here [6], and, at variance with the normalization procedure used in our time-reversal analysis [8], we do not use for the quantitiy $\operatorname{Re}\left(\epsilon_{\mathrm{L}}\right)$ the result of an external measurement.

The quantity $\alpha_{2 \pi}$ was obtained as a function of four variables, the transverse and longitudinal momentum of the charged kaon, the pion momentum and the magnetic-field polarity, and was applied as a weight to the semileptonic data, event by event. The average over all the variables is $\left\langle\alpha_{2 \pi}\right\rangle=1.12756 \pm 0.00034$.

The factor $\eta$ was determined as a function of the momentum of the pion and the electron. For its evaluation we used $\pi^{-}$and $\pi^{+}$tracks from minimum-bias data, and electrons of $\mathrm{e}^{+} \mathrm{e}^{-}$pairs selected from decays of $\mathrm{K}^{0}\left(\overline{\mathrm{~K}}^{0}\right) \rightarrow 2 \pi^{0}$, with a $\pi^{0} \rightarrow 2 \gamma$. The factor $\eta$ was applied as a weight to the semileptonic data, event by event. The value of $\eta$, averaged over the particle momenta, is $\langle\eta\rangle=1.014 \pm 0.002$. Owing to its construction, the $A_{\delta}$ asymmetry depends only weakly on $\eta$.

## 6. Background and regeneration

The data entering the asymmetry (7) were corrected for background and regeneration effects. The acceptances of the different background channels relative to the signal acceptance were evaluated using a Monte Carlo simulation [9]. The signal consists of correctly reconstructed $\mathrm{e} \pi \nu$ events and of $\mu \pi \nu$ events seen as e $\pi \nu$ ( $\approx 10 \%$ of the signal). The main background consists of residual neutral-kaon decays to $\pi^{+} \pi^{-}$and is concentrated at small decay times. At large decay times there are only contributions from $\mathrm{e} \pi \nu$ decays where the lepton and pion assignments are exchanged, and from $\pi^{+} \pi^{-} \pi^{0}$ decays. The level of these contributions remains below $1 \%$ of the signal. Using pions from a sample of minimum bias events, we estimated a background charge asymmetry of $(3 \pm 1) \%$ owing to different probabilities for a $\pi^{+}$and a $\pi^{-}$to be misidentified as positron and electron, respectively. This charge asymmetry was then included in the background evaluation.

Coherent regeneration of $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$ results arising from scattering in the material of the detector. The magnitude of this regeneration effect was determined from the value of $\Delta f=f(0)-\bar{f}(0)$, the difference in the forward-scattering amplitudes of $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$, measured by CPLEAR in a dedicated data-taking [10]. Regeneration corrections were calculated and applied on an event-by-event basis, depending on the momentum of the neutral kaon and on the position of its production and decay vertices. These corrections result in a shift of the $A_{\delta}^{\exp }$ value of the order of $0.3 \times 10^{-3}$.

## 7. Fit and systematic errors

Eq. (4), folded with the decay-time resolution [4], was fitted from 1 to $20 \tau_{\mathrm{S}}$ to the data, with $\operatorname{Re}(\delta), \operatorname{Im}(\delta)$, $\operatorname{Re}\left(x_{-}\right)$and $\operatorname{Im}\left(x_{+}\right)$as free parameters. For $\Delta m, \Gamma_{\mathrm{S}}$ and $\Gamma_{\mathrm{L}}$ we have taken the world averages of Ref. [7]. The dependence of the fit on the error of these three parameters is negligible. The result of the fit is

$$
\begin{aligned}
& \operatorname{Re}(\delta)=\left(3.0 \pm 3.3_{\text {stat }}\right) \times 10^{-4} \\
& \operatorname{Im}(\delta)=\left(-1.5 \pm 2.3_{\text {stat }}\right) \times 10^{-2} \\
& \operatorname{Re}\left(x_{-}\right)=\left(0.2 \pm 1.3_{\text {stat }}\right) \times 10^{-2} \\
& \operatorname{Im}\left(x_{+}\right)=\left(1.2 \pm 2.2_{\text {stat }}\right) \times 10^{-2}
\end{aligned}
$$

Table 1
The correlation coefficients

|  | $\operatorname{Re}(\delta)$ | $\operatorname{Im}(\delta)$ | $\operatorname{Re}\left(x_{-}\right)$ | $\operatorname{Im}\left(x_{+}\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| $\operatorname{Re}(\delta)$ | - | 0.44 | -0.56 | -0.60 |
| $\operatorname{Im}(\delta)$ | - | -0.97 | -0.91 |  |
| $\operatorname{Re}\left(x_{-}\right)$ |  | - | 0.96 |  |
| $\operatorname{Im}\left(x_{+}\right)$ |  | - |  |  |

with $\chi^{2} /$ d.o.f. $=1.14$. The correlation coefficients of the fit are shown in Table 1 . The asymmetry $A_{\delta}^{\exp }$ is plotted in Fig. 1 together with the result of the fit.

We note that $\operatorname{Re}\left(x_{-}\right)$and $\operatorname{Im}\left(x_{+}\right)$are compatible with zero, which is expected in the case where the $\Delta S=\Delta Q$ rule holds. When we fix $\operatorname{Re}\left(x_{-}\right)=\operatorname{Im}\left(x_{+}\right)=0$ in the fit, we obtain

$$
\begin{aligned}
& \operatorname{Re}(\delta)=\left(2.9 \pm 2.6_{\text {stat }}\right) \times 10^{-4} \\
& \operatorname{Im}(\delta)=\left(-0.9 \pm 2.9_{\text {stat }}\right) \times 10^{-3}
\end{aligned}
$$

that is a negligible change for $\operatorname{Re}(\delta)$, but an error of $\operatorname{Im}(\delta)$ smaller by an order of magnitude. The correlation coefficient is -0.51 .

The contributions to the systematic error from various sources,

- background level and background asymmetry,
- normalization correction,
- decay-time resolution,
- regeneration correction,
were determined, and are summarized in Table 2.
We varied by $\pm 10 \%$ the background/signal ratio of the different components of the background, and allowed the background charge asymmetry to be altered by $\pm 1 \%$ (see Section 6).

As mentioned in Section 5, the normalization factor $\alpha_{2 \pi}$ was measured with an error of $\pm 3.4 \times 10^{-4}$, and the normalization factor $\eta$ with an error of $\pm 2.0 \times 10^{-3}$.

The decay-time resolutions, evaluated by a Monte Carlo simulation [4], were let to vary in $A_{\delta}$ by $\pm 10 \%$.
Finally, the evaluation of the systematic error resulting from the regeneration correction was performed by altering the $\Delta f$ values along the one-standard-deviation ellipse in the complex plane $[\operatorname{Re}(\Delta f), \operatorname{Im}(\Delta f)]$.

From Table 2 we conclude that the main systematic error on $\operatorname{Re}(\delta)$ results from the uncertainty in the


Fig. 1. The asymmetry $A_{\delta}^{\exp }$ versus the neutral-kaon decay time (in units of $\tau_{\mathrm{S}}$ ). The solid line represents the result of the fit.

Table 2
Systematic errors

| Source | Precision | $\Delta(\operatorname{Re}(\delta))$ <br> $\left[10^{-4}\right]$ | $\Delta(\operatorname{Im}(\delta))$ <br> $\left[10^{-2}\right]$ | $\Delta\left(\operatorname{Re}\left(x_{-}\right)\right)$ <br> $\left[10^{-2}\right]$ | $\Delta\left(\operatorname{Im}\left(x_{+}\right)\right)$ <br> $\left[10^{-2}\right]$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Background level | $\pm 10 \%$ | $\pm 0.1$ | $\pm 0.1$ | $\pm 0.1$ | $\pm 0.1$ |
| Background asymmetry | $\pm 1 \%$ | $\pm 0.2$ | $\pm 0.3$ | $\pm 0.2$ | $\pm 0.3$ |
| $\alpha_{2 \pi}$ | $\pm 3.4 \times 10^{-4}$ | $\pm 0.5$ | $\pm 0.03$ | $\pm 0.02$ | $\pm 0.03$ |
| $\eta$ | $\pm 2.0 \times 10^{-3}$ | $\pm 0.02$ | $\pm 0.03$ | $\pm 0.02$ | $\pm 0.03$ |
| Decay-time resolution | $\pm 10 \%$ | negligible | $\pm 0.1$ | $\pm 0.1$ | $\pm 0.1$ |
| Regeneration | Ref. [10] | $\pm 0.25$ | $\pm 0.02$ | $\pm 0.02$ | $\pm 0.02$ |
|  |  |  |  |  |  |
| Total syst. |  | $\pm \mathbf{0 . 6}$ | $\pm \mathbf{0 . 3}$ | $\pm \mathbf{0 . 3}$ | $\pm \mathbf{0 . 3}$ |

normalization factor $\alpha_{2 \pi}$, while $\operatorname{Im}(\delta), \operatorname{Re}\left(x_{-}\right)$and $\operatorname{Im}\left(x_{+}\right)$are mainly affected by the uncertainty in the background charge asymmetry. In the case of the fit with two parameters the systematic error on $\operatorname{Re}(\delta)$ is the same while the systematic error on $\operatorname{Im}(\delta)$ becomes three times smaller.

## 8. Results and conclusions

Our final result, free of assumptions, is:

$$
\begin{aligned}
& \operatorname{Re}(\delta)=\left(3.0 \pm 3.3_{\text {stat }} \pm 0.6_{\text {syst }}\right) \times 10^{-4} \\
& \operatorname{Im}(\delta)=\left(-1.5 \pm 2.3_{\text {stat }} \pm 0.3_{\text {syst }}\right) \times 10^{-2} \\
& \operatorname{Re}\left(x_{-}\right)=\left(0.2 \pm 1.3_{\text {stat }} \pm 0.3_{\text {syst }}\right) \times 10^{-2} \\
& \operatorname{Im}\left(x_{+}\right)=\left(1.2 \pm 2.2_{\text {stat }} \pm 0.3_{\text {syst }}\right) \times 10^{-2}
\end{aligned}
$$

If we assume the validity of the $\Delta S=\Delta Q$ rule, $\operatorname{Re}\left(x_{-}\right)=\operatorname{Im}\left(x_{+}\right)=0$, our analysis yields

$$
\begin{aligned}
& \operatorname{Re}(\delta)=\left(2.9 \pm 2.6_{\text {stat }} \pm 0.6_{\text {syst }}\right) \times 10^{-4}, \\
& \operatorname{Im}(\delta)=\left(-0.9 \pm 2.9_{\text {stat }} \pm 1.0_{\text {syst }}\right) \times 10^{-3},
\end{aligned}
$$

thus improving by two and one order of magnitude, respectively, the limit obtained under the same assumptions by a re-analysis of two earlier experiments [11].

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