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K^0–$\overline{K}^0$ mass and decay-width differences: CPLEAR evaluation

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Abstract

The CPT-violation parameters Re(δ) and Im(δ) determined recently by CPLEAR are used to evaluate the K^0–$\overline{K}^0$ mass and decay-width differences, as given by the difference between the diagonal elements of the neutral-kaon mixing matrix ($M - i\Gamma/2$). The results – $(M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0}) = (-1.5 \pm 2.0) \times 10^{-18}$ GeV and $(\Gamma_{K^0K^0} - \Gamma_{\overline{K}^0\overline{K}^0}) = (3.9 \pm 4.2) \times 10^{-18}$ GeV – are consistent with CPT invariance. The CPT invariance is also shown to hold within a few times 10^{-3}–10^{-4} for many of the amplitudes describing neutral-kaon decays to different final states.

1. Introduction

The CPT theorem [1], which is based on general principles of the relativistic quantum field theory, states that any order of the triple product of the discrete symmetries C, P and T should represent an exact symmetry. The theorem predicts, among others, that particles and antiparticles have equal masses and lifetimes. The CPT symmetry has been tested in a variety of experiments (see for example Ref. [2]) and remains to date the only combination of C, P and T that is observed as an exact symmetry in nature. On the other hand there is some theoretical progress related to the string theory which may allow a consistent theoretical framework including violation of CPT to be constructed [3].

In Refs. [2,4] the mass difference between K^0 and $\overline{K}^0$ was evaluated as

$$m_{\overline{K}^0} - m_{K^0} \approx \frac{2\Delta m|\eta|\left(\frac{2}{3}\phi_{+-} + \frac{1}{3}\phi_{00} - \phi_{SW}\right)}{\sin(\phi_{SW})}. \tag{1}$$

We recall that ϕ_{+-} and ϕ_{00} are the phases of the parameters η_{+-} and η_{00} describing CP violation in the two-pion decay channel ($|\eta| = |\eta_{+-}| \approx |\eta_{00}|$); ϕ_{SW}(superweak phase) $\equiv \arctan(2\Delta m/\Delta\Gamma)$, $\Delta m = m_L - m_S$, $\Delta\Gamma = \Gamma_S - \Gamma_L$, where m_L (m_S) and Γ_L (Γ_S) are the mass and decay width, respectively, of K_L (K_S). Here we would like to stress that Eq. (1) contains the assumption of CPT invariance in the decay of neutral kaons and neglects some of the contributions from decay channels other than the two-pion [1].

[1] However, see L. Wolfenstein in Ref. [2], p.107. The relation between ϕ_{+-},ϕ_{00} and ϕ_{SW} has a long history since the seminal paper of T.T. Wu and C.N. Yang [5]. For a more recent critical discussion see Ref. [6].

In the present paper we overcome these limitations. We take advantage of the values of the CPT-violation parameters Re(δ) and Im(δ), obtained recently by CPLEAR making use of the unitarity (or Bell–Steinberger) relation [7]. The value of Im(δ) results from a variety of measurements for pionic and semileptonic decay channels, many of which are from CPLEAR, while the value of Re(δ) results essentially from the CPLEAR measurement of semileptonic decay rate asymmetries [8]. In addition to the mass difference between K^0 and $\overline{K}^0$, owing to the Re(δ) measurement, we are also able to evaluate for the first time the decay-width difference, and subsequentially analyse it in terms of individual CPT-violating decay amplitudes.

2. The neutral-kaon phenomenology

A neutral-kaon state can be written as a superposition of $|K^0\rangle$ and $|\overline{K}^0\rangle$, the eigenstates of the strong and electromagnetic interactions, with strangeness $+1$ and -1, respectively

$$|\Psi(t)\rangle = \alpha(t)|K^0\rangle + \beta(t)|\overline{K}^0\rangle. \tag{2}$$

As weak interactions do not conserve strangeness, $|K^0\rangle$ and $|\overline{K}^0\rangle$ undergo strangeness oscillations as well as decays. The time evolution of the state in Eq. (2) is described by

$$\frac{d}{dt}\Psi = -i\Lambda\Psi,$$

$$\Lambda \equiv \begin{pmatrix} \Lambda_{K^0K^0} & \Lambda_{K^0\overline{K}^0} \\ \Lambda_{\overline{K}^0K^0} & \Lambda_{\overline{K}^0\overline{K}^0} \end{pmatrix} \equiv M - \frac{i}{2}\Gamma$$

$$\equiv \begin{pmatrix} M_{K^0K^0} & M_{K^0\overline{K}^0} \\ M_{\overline{K}^0K^0} & M_{\overline{K}^0\overline{K}^0} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{K^0K^0} & \Gamma_{K^0\overline{K}^0} \\ \Gamma_{\overline{K}^0K^0} & \Gamma_{\overline{K}^0\overline{K}^0} \end{pmatrix}, \tag{3}$$

where M and Γ are Hermitian matrices known as the mass and decay matrices. The eigenvalues corresponding to the physical states $|K_L\rangle$ and $|K_S\rangle$ are $\lambda_{L,S} = m_{L,S} - \frac{i}{2}\Gamma_{L,S}$, with $\Delta m \approx 2|M_{K^0\overline{K}^0}|$ and $\Delta\Gamma \approx 2|\Gamma_{K^0\overline{K}^0}|$. The symmetry properties of the matrix elements are shown in Table 1, together with the parameters ϵ and δ commonly used to describe, in the time evolution, the breaking of the symmetries CPT, T and CP [8–12] [2]:

$$\epsilon = \frac{\Lambda_{\overline{K}^0K^0} - \Lambda_{K^0\overline{K}^0}}{2(\lambda_L - \lambda_S)}, \quad \delta = \frac{\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0}}{2(\lambda_L - \lambda_S)}. \quad (4)$$

2.1. CPT test of the mixing matrix

The parameter δ,

$$\delta = |\delta|\exp\left[i\left(\phi_{SW} - \phi_{CPT} - \frac{\pi}{2}\right)\right],$$

with

$$\phi_{CPT} = \arctan\left(\frac{1}{2}\frac{(\Gamma_{\overline{K}^0\overline{K}^0} - \Gamma_{K^0K^0})}{(M_{\overline{K}^0\overline{K}^0} - M_{K^0K^0})}\right),$$

is conveniently represented in the complex plane [11] by the projections along the ϕ_{SW} axis ($\delta_{\parallel}$) and its normal ($\delta_{\perp}$):

$$\delta_{\parallel} = \frac{1}{4}\frac{\Gamma_{K^0K^0} - \Gamma_{\overline{K}^0\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\Gamma}{2}\right)^2}},$$

$$\delta_{\perp} = \frac{1}{2}\frac{M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\Gamma}{2}\right)^2}}. \quad (5)$$

The parameters $\delta_{\parallel}$ and $\delta_{\perp}$ can be expressed as functions of the measured quantities $\mathrm{Re}(\delta)$, $\mathrm{Im}(\delta)$ and ϕ_{SW} as

$$\delta_{\parallel} = \mathrm{Re}(\delta)\cos(\phi_{SW}) + \mathrm{Im}(\delta)\sin(\phi_{SW}),$$

$$\delta_{\perp} = -\mathrm{Re}(\delta)\sin(\phi_{SW}) + \mathrm{Im}(\delta)\cos(\phi_{SW}), \quad (6)$$

[2] The parametrizations presented in Ref. [12] are equivalent to ours but formulated in a slightly different notation.

Table 1
The properties of the Λ-matrix elements under the assumption of CPT, T and CP invariance and the parameters which describe the breaking of these symmetries

Symmetry	Λ-matrix properties	Parameters
CPT	$\Lambda_{K^0K^0} = \Lambda_{\overline{K}^0\overline{K}^0}$	δ
T	$\lvert\Lambda_{K^0\overline{K}^0}\rvert = \lvert\Lambda_{\overline{K}^0K^0}\rvert$	ϵ
CP	$\Lambda_{K^0K^0} = \Lambda_{\overline{K}^0\overline{K}^0}$,	$\epsilon_L = \epsilon - \delta$,
	$\lvert\Lambda_{K^0\overline{K}^0}\rvert = \lvert\Lambda_{\overline{K}^0K^0}\rvert$	$\epsilon_S = \epsilon + \delta$

and allow in turn the K^0–$\overline{K}^0$ decay-width and mass differences to be determined as

$$\Gamma_{K^0K^0} - \Gamma_{\overline{K}^0\overline{K}^0} = \delta_{\parallel}\frac{2\Delta\Gamma}{\cos(\phi_{SW})},$$

$$M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0} = \delta_{\perp}\frac{\Delta\Gamma}{\cos(\phi_{SW})}. \quad (7)$$

Thus the evaluation of the K^0–$\overline{K}^0$ mass and decay-width differences is straightforward, once the CPT-violation parameters $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ are known.

2.2. CPT test of the decay amplitudes

K^0 ($\overline{K}^0$) decays to a specified final state f occur with an amplitude A_f ($\overline{A}_f$). By assuming unitarity [13], the elements of the Γ-matrix are given by the K^0 ($\overline{K}^0$) decay amplitudes to real final states f, with

$$\Gamma_{K^0K^0} = \sum A_f^* A_f, \quad \Gamma_{\overline{K}^0\overline{K}^0} = \sum \overline{A}_f^* \overline{A}_f,$$

$$\Gamma_{K^0\overline{K}^0} = \Gamma_{\overline{K}^0K^0}^* = \sum A_f^* \overline{A}_f. \quad (8)$$

The elements of the M-matrix contain instead also the transition amplitudes to all virtual states. This implies that CPT violation could manifest to first order in the M-matrix, but only to a higher order in the Γ-matrix.

The decay amplitudes A_f and $\overline{A}_f$ are parametrized to account for selection rules based on discrete symmetries, isospin changes (for pionic decays) or the $\Delta S = \Delta Q$ rule (for semileptonic decays). For the T (transition matrix) elements of two-pion final states we write [10–12]

$$\langle\pi\pi, I|T|K^0\rangle = (A_I + B_I)e^{i\delta_I},$$

$$\langle\pi\pi, I|T|\overline{K}^0\rangle = (A_I^* - B_I^*)e^{i\delta_I}, \quad I = 0, 2,$$

where A_I and B_I are CPT symmetric and antisymmetric amplitudes, respectively, and δ_I are the $\pi\pi$ phase-shifts of channels with total isospin I. For the three-pion final states, the isospin values range from

Table 2
Experimental values of K_S and K_L parameters [2,7] used in the present analysis

Parameter	Value
$1/\Gamma_S$	$(0.8934 \pm 0.0008) \times 10^{-10} \hbar^{-1}$ s
$1/\Gamma_L$	$(5.17 \pm 0.04) \times 10^{-8} \hbar^{-1}$ s
$\Delta\Gamma$	$(1117.4 \pm 1.0) \times 10^{7} \hbar$ s^{-1} $= (7.355 \pm 0.007) \times 10^{-15}$ GeV
Δm	$(530.2 \pm 1.5) \times 10^{7} \hbar$ s^{-1} $= (3.490 \pm 0.010) \times 10^{-15}$ GeV

0 to 3, with $I = 1$ being the most favoured one [14]. For the purpose of this paper we write simply

$$\langle \pi\pi\pi, I | T | K^0 \rangle = (A_1 + B_1) e^{i\delta_1},$$

$$\langle \pi\pi\pi, I | T | \overline{K}^0 \rangle = (A_1^* - B_1^*) e^{i\delta_1}.$$

Finally, four decay amplitudes describe semileptonic decays [11,12]:

$$\langle \ell^+ \pi^- \nu | T | K^0 \rangle = a + b,$$

$$\langle \ell^- \pi^+ \overline{\nu} | T | \overline{K}^0 \rangle = a^* - b^*,$$

$$\langle \ell^- \pi^+ \overline{\nu} | T | K^0 \rangle = c + d,$$

$$\langle \ell^+ \pi^- \nu | T | \overline{K}^0 \rangle = c^* - d^*,$$

with $\ell^{\pm} = e^{\pm}$, $\mu^{\pm}$. Here, Re(a) is T and CPT symmetric and all imaginary parts are T violating; c and d describe $\Delta S = -\Delta Q$ decays, and b and d are CPT violating.

3. Results on mass and decay-width differences

CPLEAR has recently obtained for Re(δ) and Im(δ) the values [7]

$$\text{Re}(\delta) = (2.4 \pm 2.8) \times 10^{-4},$$

$$\text{Im}(\delta) = (2.4 \pm 5.0) \times 10^{-5}, \quad (9)$$

with a correlation coefficient of 5%. Using these values and the values for $\Delta\Gamma$ and Δm of Table 2, we obtain from Eq. (6)

$$\delta_{\parallel} = (1.9 \pm 2.0) \times 10^{-4},$$

$$\delta_{\perp} = (-1.5 \pm 2.0) \times 10^{-4}, \quad (10)$$

and subsequently from Eq. (7)

$$\Gamma_{K^0K^0} - \Gamma_{\overline{K}^0\overline{K}^0} = (3.9 \pm 4.2) \times 10^{-18} \text{ GeV},$$

$$M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0} = (-1.5 \pm 2.0) \times 10^{-18} \text{ GeV}, \quad (11)$$

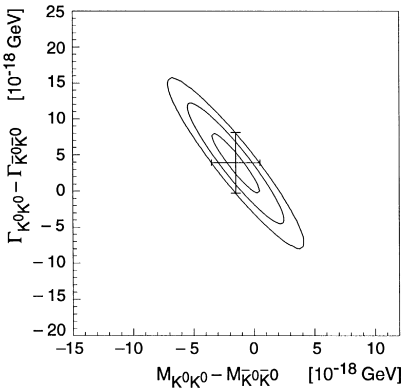

Fig. 1. The K^0–$\overline{K}^0$ decay-width versus mass difference. The 1σ, 2σ and 3σ ellipses are also shown.

with a correlation coefficient of -0.95. Fig. 1 shows the error ellipses corresponding to 1σ, 2σ and 3σ. Our result on the mass difference is a factor of two better than the one obtained with a similar calculation in Ref. [15]. We note that the improvement is mainly due to Re(δ) being now known with a smaller error, see Eq. (9).

The error of Re(δ) becomes even smaller if we assume CPT-invariant decay amplitudes, that is $\Gamma_{K^0K^0} = \Gamma_{\overline{K}^0\overline{K}^0}$ or, equivalently, Re$(\delta) = -$Im$(\delta) \times \tan(\phi_{SW})$. In this case Re$(\delta)$ can be determined by Im(δ) and the parameter $\delta_{\perp}$ becomes $\delta_{\perp} =$ Im$(\delta)/\cos(\phi_{SW})$. The results for $M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0}$ are shown in Table 3 depending on the values for Im(δ) which are obtained from the unitarity relation under different conditions: a) no restriction [7], b) equal CP-violation parameters for the decay to $\pi^0\pi^0\pi^0$ and to $\pi^+\pi^-\pi^0$, i.e. $\eta_{000} = \eta_{+-0}$ [7], c) only the $\pi\pi$ decay channel contributes to the unitarity relation.

Table 3
Mass difference assuming $\Gamma_{K^0K^0} - \Gamma_{\overline{K}^0\overline{K}^0} = 0$: values and modulus limits at 90% CL for different values of Im(δ) (see text)

Condition	Im(δ) [10^{-5}]	$(M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0})$ [10^{-19}GeV]	$\lvert M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0}\rvert$ [10^{-19}GeV]
a)	2.4 ± 5.0	3.3 ± 7.0	≤ 12.7
b)	-0.5 ± 2.0	-0.7 ± 2.8	≤ 4.8
c)	-0.1 ± 1.9	-0.1 ± 2.7	≤ 4.4

4. Remark on the method

We shall now compare the method outlined above with the one leading to Eq. (1). With the notation of Section 2 for the decay amplitudes, we obtain from the η_{+-} and η_{00} definitions [11]

$$\eta_{+-} = \epsilon - \delta + \left(\mathrm{i}\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} + \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} \right) + \epsilon' , \tag{12a}$$

$$\eta_{00} = \epsilon - \delta + \left(\mathrm{i}\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} + \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} \right) - 2\epsilon' , \tag{12b}$$

$$\epsilon' = \frac{1}{\sqrt{2}} \mathrm{e}^{\mathrm{i}(\delta_2 - \delta_0)} \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} \left[\mathrm{i}\left(\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right) + \left(\frac{\mathrm{Re}(B_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} \right) \right] . \tag{12c}$$

The set of Eqs.(12) is visualized in Fig. 2. For this representation we have used the T-violation parameter $\epsilon_T = \epsilon - \mathrm{i}\,\mathrm{Im}(\Gamma_{K^0\overline{K}^0})/\Delta\Gamma$, which has a phase equal to ϕ_{SW} [11]. We have also introduced the quantity $\Delta\phi = \mathrm{Im}(\Gamma'_{K^0\overline{K}^0})/\Delta\Gamma$ which stands for one half the phase of the off-diagonal Γ-matrix element $\Gamma'_{K^0\overline{K}^0}$ corresponding to neutral kaons decaying to channels other than two-pion ($I=0$) state. From Eqs. (12) we obtain expressions for Im(δ), Re(δ) and, together with Eq. (6), for $\delta_\perp$:

$$\mathrm{Im}(\delta) = \cos(\phi_{SW})\,|\eta_{+-}|\left(\phi_{SW} - \tfrac{2}{3}\phi_{+-} - \tfrac{1}{3}\phi_{00}\right) + \Delta\phi , \tag{13a}$$

$$\mathrm{Re}(\delta) = -\sin(\phi_{SW})\,|\eta_{+-}|\left(\phi_{SW} - \tfrac{2}{3}\phi_{+-} - \tfrac{1}{3}\phi_{00}\right) + \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} , \tag{13b}$$

$$\delta_\perp = |\eta_{+-}|\left(\phi_{SW} - \tfrac{2}{3}\phi_{+-} - \tfrac{1}{3}\phi_{00}\right) + \Delta\phi\cos(\phi_{SW}) - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)}\sin(\phi_{SW}) . \tag{13c}$$

Here, use is made of the fact that $|\eta_{+-}| \approx |\eta_{00}| \approx |\epsilon_T|$. This approximation is no longer appropriate when computing $\delta_\parallel$ for which we obtain

$$\delta_\parallel \approx |\epsilon_T| - |\eta_{+-}| + \Delta\phi\sin(\phi_{SW}) + \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)}\cos(\phi_{SW}) . \tag{14}$$

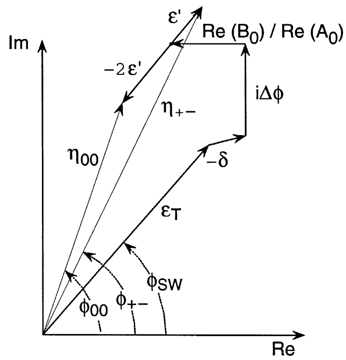

Fig. 2. CP-, T- and CPT-violation parameters when a neutral kaon decays to $\pi\pi$.

Owing to the lack of precise information on $|\epsilon_T| - |\eta_{+-}|$, one evaluates only $\delta_\perp$ (and the mass difference), without any explicit consideration of $\delta_\parallel$ (and the decay-width difference). Finally, when the terms containing $\Delta\phi$ and $\mathrm{Re}(B_0)/\mathrm{Re}(A_0)$ are neglected [3] Eqs. (13), (7) reduce to Eq. (1) and to case c) of Table 3, leading to the limit $|M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0}| \leq 4.4 \times 10^{-19}$ GeV (90% CL).

With a similar approach, one could also simply use Eqs. (12) and neglect as well, in addition to $\Delta\phi$ and $\mathrm{Re}(B_0)/\mathrm{Re}(A_0)$, the projection of ϵ' along the normal to the ϕ_{SW} axis, $\epsilon'_\perp = \frac{1}{3}|\eta_{+-}|(\phi_{+-} - \phi_{00})$ – which means to neglect the real part of both the CPT-violating amplitudes B_0 and B_2, see Section 5. This procedure leads to a slightly lower limit for $|M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0}|$, that is $|M_{K^0K^0} - M_{\overline{K}^0\overline{K}^0}| \leq 4.0 \times 10^{-19}$ (90% CL). For these evaluations we have used the values entered in our unitarity analysis [13], that is $|\eta_{+-}| = (2.283 \pm 0.025) \times 10^{-3}$, $(\phi_{00} - \phi_{+-}) = (-0.3 \pm 0.8)°$ and $\phi_{+-} = (43.6 \pm 0.6)°$ [2,18].

[3] The measurements of CPLEAR in semileptonic and 3π sectors have allowed to set stringent limits on $\Delta\phi$. If one assumes the $I=1$ decay amplitude to be dominant in the three-pion decay so that $\eta_{000} = \eta_{+-0}$, we obtain $\Delta\phi = (-5.8 \pm 8.1) \times 10^{-6}$ and $\delta_\perp = (-0.4 \pm 2.7) \times 10^{-5}$, while $\delta_\perp = (-0.0 \pm 2.6) \times 10^{-5}$ for $\Delta\phi = 0$. If one uses the measured value for η_{000}, the error on $\Delta\phi$ increases by an order of magnitude and becomes dominant in Eq. (13b), provided that $\mathrm{Re}(B_0)/\mathrm{Re}(A_0)$ can be neglected. Without this last restriction the error of $\delta_\perp$ becomes as large as $\approx 2 \times 10^{-4}$, see Section 5.

5. Results on CPT-violating decay amplitudes

The limit on the decay-width difference obtained from Eq. (11) represents a global evaluation of a possible CPT violation in the decay. However, we may also give some information on the individual CPT-violating decay amplitudes.

For the semileptonic decays, the parameters Re(y) and Re(x_-), $y = -b/a$ and $x_- = -d^*/a$, describing CPT violation in $\Delta S = \Delta Q$ and $\Delta S \neq \Delta Q$ transitions, respectively, and their sum have already been determined in Ref. [7],

$$\mathrm{Re}(y) = (0.3 \pm 3.1) \times 10^{-3},$$
$$\mathrm{Re}(x_-) = (-0.5 \pm 3.0) \times 10^{-3},$$
$$\mathrm{Re}(y + x_-) = (-2.0 \pm 3.0) \times 10^{-4}.$$

For the pionic decays, only the parameters $\mathrm{Re}(B_I)/\mathrm{Re}(A_I)$ are estimated. With no attempt to make a global fit to the data, to perform this estimation we express the K^0–$\overline{K}^0$ decay-width difference, according to its definition, as

$$\frac{\Gamma_{K^0K^0} - \Gamma_{\overline{K}^0\overline{K}^0}}{2\Gamma_S} = \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} + \left|\frac{A_2}{A_0}\right|^2 \frac{\mathrm{Re}(B_2)}{\mathrm{Re}(A_2)} + \frac{\Gamma_L}{\Gamma_S}\left[\mathrm{BR}(K_L \to 3\pi)\frac{\mathrm{Re}(B_1)}{\mathrm{Re}(A_1)} - 2\mathrm{BR}(K_L \to \ell\pi\nu)\mathrm{Re}(y)\right], \quad (15)$$

where BR stands for branching ratio. In Eq. (15), the left-hand side is determined from Eq. (11) to be $(2.6 \pm 2.9) \times 10^{-4}$. On the right-hand side, the last term is estimated to be $\approx 5 \times 10^{-6}$ with the values of the branching ratio from Ref. [2], Re(y) as given above, and the measured value of $\mathrm{Re}(\eta_{+-0})$ [14] considered as an upper limit for $\mathrm{Re}(B_1)/\mathrm{Re}(A_1)$. We are then left with the possible contributions to the decay-width difference from the two-pion decay channel.

Since $(\delta_2 - \delta_0) = -(42 \pm 4)^\circ$ [16] and $\phi_{SW} = -(43.50 \pm 0.08)^\circ$ (with the values of Table 2), we obtain with a good approximation from Eqs. (12)

$$\tfrac{1}{3}(\phi_{00} - \phi_{+-})|\eta_{+-}| = \frac{1}{\sqrt{2}}\frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)}\left[\frac{\mathrm{Re}(B_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)}\right]. \quad (16)$$

We estimate $\mathrm{Re}(A_2)/\mathrm{Re}(A_0) \approx |A_2/A_0| \approx 0.04479 \pm 0.00020$ [17], with $\mathrm{Re}(B_2)/\mathrm{Re}(A_2)$ and $\mathrm{Re}(B_0)/\mathrm{Re}(A_0) \ll 1$; we also take $|\eta_{+-}| = (2.283 \pm 0.025) \times 10^{-3}$ and $(\phi_{00} - \phi_{+-}) = (-0.3 \pm 0.8)^\circ$ [2,18]. By using these values for $\mathrm{Re}(A_2)/\mathrm{Re}(A_0)$, $|\eta_{+-}|$ and $(\phi_{00} - \phi_{+-})$ we obtain from Eq. (15)

$$0.002 \times \frac{\mathrm{Re}(B_2)}{\mathrm{Re}(A_2)} + \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} = (2.6 \pm 2.9) \times 10^{-4},$$

and from Eq. (16)

$$\frac{\mathrm{Re}(B_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} = (-1.3 \pm 3.4) \times 10^{-4},$$

hence,

$$\frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)} = (2.6 \pm 2.9) \times 10^{-4},$$

$$\frac{\mathrm{Re}(B_2)}{\mathrm{Re}(A_2)} = (1.3 \pm 4.5) \times 10^{-4}.$$

6. Conclusion

We have analysed in detail possible CPT-violating contributions both to the K^0–$\overline{K}^0$ mixing matrix and to the individual amplitudes for K^0 and $\overline{K}^0$ decays to $\pi\pi$ and to $\ell\pi\nu$. Without any assumption, the K^0–$\overline{K}^0$ mass and decay-width differences are shown to be consistent with CPT invariance within a few 10^{-18} GeV. The measurement of the decay-width difference relies mainly on the CPLEAR measurements of the semileptonic decay rates and the parameter Re(δ) [8], also allowing possible cancellation effects to be disentangled. The ratio between CPT-violating and CPT-invariant amplitudes is shown to be smaller than a few times 10^{-3}–10^{-4} for a number of cases.

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