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A new determination of the $K_L - K_S$ mass difference and the phase of the CP violation parameter η_{+-} from an evaluation of experimental data

CPLEAR Collaboration

R. Adler[2], A. Angelopoulos[1], A. Apostolakis[1], E. Aslanides[11], G. Backenstoss[2], C.P. Bee[9], O. Behnke [17], A. Benelli[9], V. Bertin[11], F. Blanc[7,13], P. Bloch[4], P. Carlson[15], M. Carroll[9], J. Carvalho[5], E. Cawley[9], S. Charalambous[16], G. Chardin[14], M.B. Chertok[3], A. Cody[9], M. Danielsson[15], S. Dedoussis[16], M. Dejardin[14], J. Derre[14], A. Ealet[11], B. Eckart[2], C. Eleftheriadis[16], I. Evangelou[8], L. Faravel [7], P. Fassnacht[11], C. Felder[2], R. Ferreira-Marques[5], W. Fetscher[17], M. Fidecaro[4], A. Filipčič[10], D. Francis[3], J. Fry[9], E. Gabathuler[9], R. Gamet[9], D. Garreta[14], H.-J. Gerber[17], A. Go[3,15], C. Guyot[14], A. Haselden[9], P.J. Hayman[9], F. Henry-Couannier[11], R.W. Hollander[6], E. Hubert[11], K. Jon-And[15], P.-R. Kettle[13], C. Kochowski[14], P. Kokkas[2], R. Kreuger[6], R. Le Gac[11], F. Leimgruber[2], A. Liolios[16], E. Machado[5], I. Mandić[10], N. Manthos[8], G. Marel[14], M. Mikuž[10], J. Miller[3], F. Montanet[11], T. Nakada[13], B. Pagels[17], I. Papadopoulos[16], P. Pavlopoulos[2], J. Pinto da Cunha[5], A. Policarpo[5], G. Polivka[2], R. Rickenbach[2], B.L. Roberts[3], T. Ruf[4], L. Sakeliou[1], P. Sanders[9], C. Santoni[2], M. Schäfer[17], L.A. Schaller[7], T. Schietinger[2], A. Schopper[4], P. Schune[14], A. Soares[14], L. Tauscher[2], C. Thibault[12], F. Touchard[11], C. Touramanis[4], F. Triantis[8], E. Van Beveren[5], C.W.E. Van Eijk[6], G. Varner[3], S. Vlachos[2], P. Weber[17], O. Wigger[13], M. Wolter[17], C. Yeche[14], D. Zavrtanik[10] and D. Zimmerman[3].

Abstract

Any measurement of the phase φ_{+-} of the CP violation parameter η_{+-} is correlated with the value of the $K_L - K_S$ mass difference Δm. This correlation differs for different experiments and has to be taken into account when determining average values. From an analysis including recent measurements, we obtain the average values $< \Delta m >= (530.6 \pm 1.3) \times 10^7 \hbar/\mathrm{s}$ and $< \varphi_{+-} >= 43.75^\circ \pm 0.60^\circ$. This value of φ_{+-} is in good agreement with the superweak phase $\varphi_{SW} = 43.44^\circ \pm 0.09^\circ$.

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1 Introduction

The comparison of the phase φ_{+-} of the CP violation parameter η_{+-} with the superweak phase $\varphi_{SW} \equiv \tan^{-1}(2\Delta m/\Delta\Gamma)$, Δm, $\Delta\Gamma$ being the difference between the K_L and K_S mass and total width respectively, yields the world's best limits of CPT violation [1, 2]. The phase φ_{+-} has been measured by many experiments over the last 30 years. Its experimental value has a different dependence on Δm for different experiments. The values of the most precise experiments [3–6], adopted by PDG [1], as well as the values of two recent experiments [7, 8] are presented in Table 1 together with their quoted Δm and τ_S dependence. In order to combine these measurements, the likelihood distribution as a function of the parameters φ_{+-}, Δm and the K_S lifetime τ_S has to be known for each experiment.

A gaussian likelihood distribution taking into account the correlation between φ_{+-} and Δm can be defined with the published information for the experiments [6, 7, 8]. The authors of the other experiments [3–5] simply state that their value of φ_{+-} is strongly correlated with Δm and give a linear dependence. In order to define a gaussian likelihood distribution also for these experiments, we have to make some assumptions about the correlation between φ_{+-} and Δm and about the central value for Δm. For experiment [3], which gives the most precise value of φ_{+-} among these three experiments, we have been able to deduce that the correlation is larger than 99% [9]. We assume this correlation also holds for the other two experiments [4, 5]. For such a strong correlation between φ_{+-} and Δm, the error ellipse of the individual experiment is degenerated to a band (see Figure 1) within the limit of Δm defined by the other experiments [6–8, 10–13]. Consequently the experiments [3–5] can contribute to the fit only as one degree of freedom. The systematic uncertainty for the average values due to our assumptions is estimated by varying the correlation between φ_{+-} and Δm and the central value of Δm.

The world average value of τ_S [1] is dominated by experiment [6]. The authors assume in their fit $\varphi_{+-} = \varphi_{SW}$. A difference of 1° between φ_{+-} and φ_{SW} results only in a shift in their measurement of τ_S [6] of 0.0008×10^{-10} s. Without more information from the other experiments, we use the world average value of τ_S and take into account the linear dependence of φ_{+-} with τ_S for each experiment if given (Table 1) in our fitting procedure. Since τ_S is known to a better precision than Δm, the τ_S dependence of φ_{+-} is less important. For the same reason, the change in the experimental value of Δm, when varying the value of τ_S within its error, is negligible (Table 2).

1) University of Athens, Greece
2) University of Basle, Switzerland
3) Boston University, USA
4) CERN, Geneva, Switzerland
5) LIP and University of Coimbra, Portugal
6) Technical University of Delft, Netherlands
7) University of Fribourg, Switzerland
8) University of Ioannina, Greece
9) University of Liverpool, UK
10) J. Stefan Inst. and Dep. of Physics, University of Ljubljana, Slovenia
11) CPPM, IN2P3-CNRS et Université d'Aix-Marseille II, Marseille, France
12) CSNSM, IN2P3-CNRS, Orsay, France
13) Paul-Scherrer-Institut, Villigen, Switzerland
14) CEA, DSM/DAPNIA, CE Saclay, France
15) Royal Institute of Technology Stockholm, Sweden
16) University of Thessaloniki, Thessaloniki, Greece
17) ETH-IPP Zürich, Switzerland.

2 Description of the fits

We assume that the measured values $X_i = (\varphi_{+-}{}^i, \Delta m^i)$ are distributed around the true value X according to a gaussian likelihood distribution:

$$L_i(X) \quad = \quad k_i e^{-\frac{1}{2}(X_i - X)C_i^{-1}(X_i - X)^T}, \tag{1}$$

where C_i is the corresponding covariance matrix:

$$C_i = \begin{pmatrix} \sigma^i_\varphi & c^i_{\varphi\Delta m} \\ c^i_{\varphi\Delta m} & \sigma^i_{\Delta m} \end{pmatrix} , \tag{2}$$

and k_i is a normalisation factor which generally depends on the elements of C_i. In the case of experiments [10-13], $X_i = \Delta m^i$. The combined likelihood distribution of all measurements is then given by:

$$\mathcal{L}(X) = \prod_i L_i(X) \ . \tag{3}$$

The best estimate of X and its error is given by minimizing $\mathcal{L}(X)$, i.e. minimizing $\chi^2(X) = \sum(X_i - X)^T C_i^{-1}(X_i - X)$ with respect to X. Errors common to different experiments can be taken into account in this procedure by expanding X_i and C_i as follows:

$$X_i \rightarrow X_i = \begin{pmatrix} X_k \\ X_l \end{pmatrix} \qquad C_i \rightarrow C_i = \begin{pmatrix} C_k & \sigma^2 \\ \sigma^2 & C_l \end{pmatrix} , \tag{4}$$

where σ is the error common to experiment k and l.

Most of the experiments quote the value of φ_{+-} for a fixed value $\Delta m = \Delta\tilde{m}$ together with the linear dependence on Δm. We recall that for a fixed value $\Delta\tilde{m}$ the value of φ_{+-} and its error, $\tilde{\varphi}_{+-}$ and $\tilde{\sigma}_\varphi$ respectively, are given by:

$$\tilde{\varphi}_{+-} \quad = \quad \varphi_{+-} + \rho\frac{\sigma_\varphi}{\sigma_{\Delta m}}\left(\Delta\tilde{m} - \Delta m\right) \tag{5}$$

$$\tilde{\sigma}_\varphi \quad = \quad \sigma_\varphi\sqrt{1 - \rho^2} \tag{6}$$

where φ_{+-}, σ_φ and Δm, $\sigma_{\Delta m}$ are the central values and errors of a two parameter fit and ρ is the correlation coefficient between φ_{+-} and Δm. Therefore we are able to reconstruct the full covariance matrix by using the linear dependence (Table 1), the value of Δm and its error (Table 2).

2.1 Average values for φ_{+-} and Δm

The data used as input to our fits is summarized in Table 1 and Table 2 and also presented together with the final result in Figure 1. Since the values of φ_{+-} depend on the value of τ_S, we leave τ_S as an additional free parameter in the fit. However its value is constrained to the world average of $< \tau_S >= (0.8926 \pm 0.0012) \times 10^{-10}$ s [1].

Using as input to our fit the experimental data available up to 1994, we find similar results as PDG [1] although our error for $< \Delta m >$ is smaller compared to the PDG result due to the additional information used. For Δm we find $< \Delta m >= (532.2 \pm 1.8 \pm 0.1) \times 10^7 \hbar/\mathrm{s}$ and for φ_{+-} the value $< \varphi_{+-} >= 44.5^\circ \pm 1.0^\circ \pm 0.1^\circ$, where the second error reflects the uncertainty of the correlation between φ_{+-} and Δm for the experiments [3–5]. Two experiments [6, 10] give a large contribution to the χ^2, the first giving a low value and the second a high value of Δm

compared to the average value. The χ^2/degree of freedom (dof) for the combined average of φ_{+-} and Δm is 1.0, and in contrast to PDG we do not need to scale the error of Δm by 1.2.

If we include the three recently published measurements [7, 8, 13] of Δm and φ_{+-} we find the results shown in Table 3 (Fit A). The value of Δm drops by 1 σ and only one experiment [10] now gives a large contribution to the χ^2. Since this experiment deviates only by 2σ and the total χ^2/dof is 0.8, we have retained it in our fit. By using only the experiments [6–8, 10–13], which give the full covariance matrix of their measurements, we obtain the results shown in Table 3 (Fit B), which are in good agreement with Fit A.

3 Conclusion

Averaging Δm and φ_{+-} independently is not an adequate method as these two parameters are correlated. We have evaluated both parameters, Δm and φ_{+-} by taking into account the correlations. Our final result using the experiments [3–8,10–13] is:

$$\varphi_{+-} = 43.75^\circ \pm 0.60^\circ \tag{7}$$
$$\Delta m = (530.6 \pm 1.3) \times 10^7 \hbar/\mathrm{s}\ , \tag{8}$$

with a correlation coefficient of 0.74 between φ_{+-} and Δm, and -0.30 between φ_{+-} and τ_S, i.e.:

$$\varphi_{+-} = 43.75^\circ \pm 0.39^\circ + 0.321\,(\Delta m - 530.6)^\circ - 180\,(\tau_S - 0.8923)^\circ\ . \tag{9}$$

The value of φ_{+-} is in good agreement with the superweak phase $\varphi_{SW} = 43.44^\circ \pm 0.09^\circ$ as expected from CPT invariance.

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Table 1: Most precise measurements of φ_{+-}. For the experiments [6,7] we assume a common systematic error of $\pm 0.3°$ due to regeneration uncertainties.

Experiment	φ_{+-}	correlation $\varphi_{+-}/\Delta m$
Geweniger [3]	$49.4° \pm 1.0° + 0.565\,(\Delta m - 540.0)°$	> 0.99
Carithers [4]	$45.5° \pm 2.8° + 0.224\,(\Delta m - 534.8)°$	unknown
Carosi [5]	$46.9° \pm 1.6° + 0.579\,(\Delta m - 535.1)° + 303\,(\tau_S - 0.8922)°$	unknown
† E731 [6]	$42.21° \pm 0.9° + 0.189\,(\Delta m - 525.7)° - 460\,(\tau_S - 0.8922)°$	0.662
E773 [7]	$43.53° \pm 0.8° + 0.173\,(\Delta m - 528.2)° - 275\,(\tau_S - 0.8926)°$	0.613
CPLEAR [8]	$42.7° \pm 1.1° + 0.316\,(\Delta m - 527.4)° + 30\,(\tau_S - 0.8926)°$	0.919

† The 1994 PDG [1] quotation has an error in the central value for the Δm dependence

Table 2: Most precise measurements of Δm. The experiments [10–13] measure Δm independently of φ_{+-}, whereas the experiments [6–8] obtain Δm from a fit with floating Δm and φ_{+-}. The experiments [11] and [12] have a common systematic error of $\pm 1.5 \times 10^7 \hbar/\mathrm{s}$.

Experiment	$\Delta m [10^7 \hbar/\mathrm{s}]$
E731 [6]	525.7 ± 4.9
E773 [7]	529.7 ± 3.7
CPLEAR [8]	529.5 ± 6.7
Cullen [10]	542.0 ± 6.0
Gjesdal [11]	$533.4 \pm 4.0 \pm 1.5$
Geweniger [12]	$534.0 \pm 2.55 \pm 1.5 + 0.02\left(\frac{\tau_S - 0.8994}{0.0012}\right)$
CPLEAR [13]	$527.4 \pm 2.9 \pm 0.10\left(\frac{\tau_S - 0.8926}{0.0012}\right)$

Table 3: Results from Fit A and Fit B. Fit A is made using all the experiments whereas Fit B is based only on the experiments which give the full covariance matrix of their measurements. The second error for Fit A is obtained by varying the correlation between φ_{+-} and Δm for the older experiments [3–5].

Parameter	Fit A	Fit B
$\Delta m [10^7 \hbar \mathrm{s}^{-1}]$	$530.6 \pm 1.3 \pm 0.1$	530.8 ± 1.5
φ_{+-}	$43.75° \pm 0.60° \pm 0.10°$	$43.67° \pm 0.62°$
$\tau_S\ [10^{-10}\,\mathrm{s}^{-1}]$	0.8923 ± 0.0010	0.8925 ± 0.0011
χ^2/dof	0.81	1.03
$\varphi_{SW}^{\dagger}$	$43.44° \pm 0.09°$	$43.45° \pm 0.09°$

† For the $\mathrm{K_L}$ lifetime we used $\tau_L = (5.15 \pm 0.04) \times 10^{-8}$ s [1].

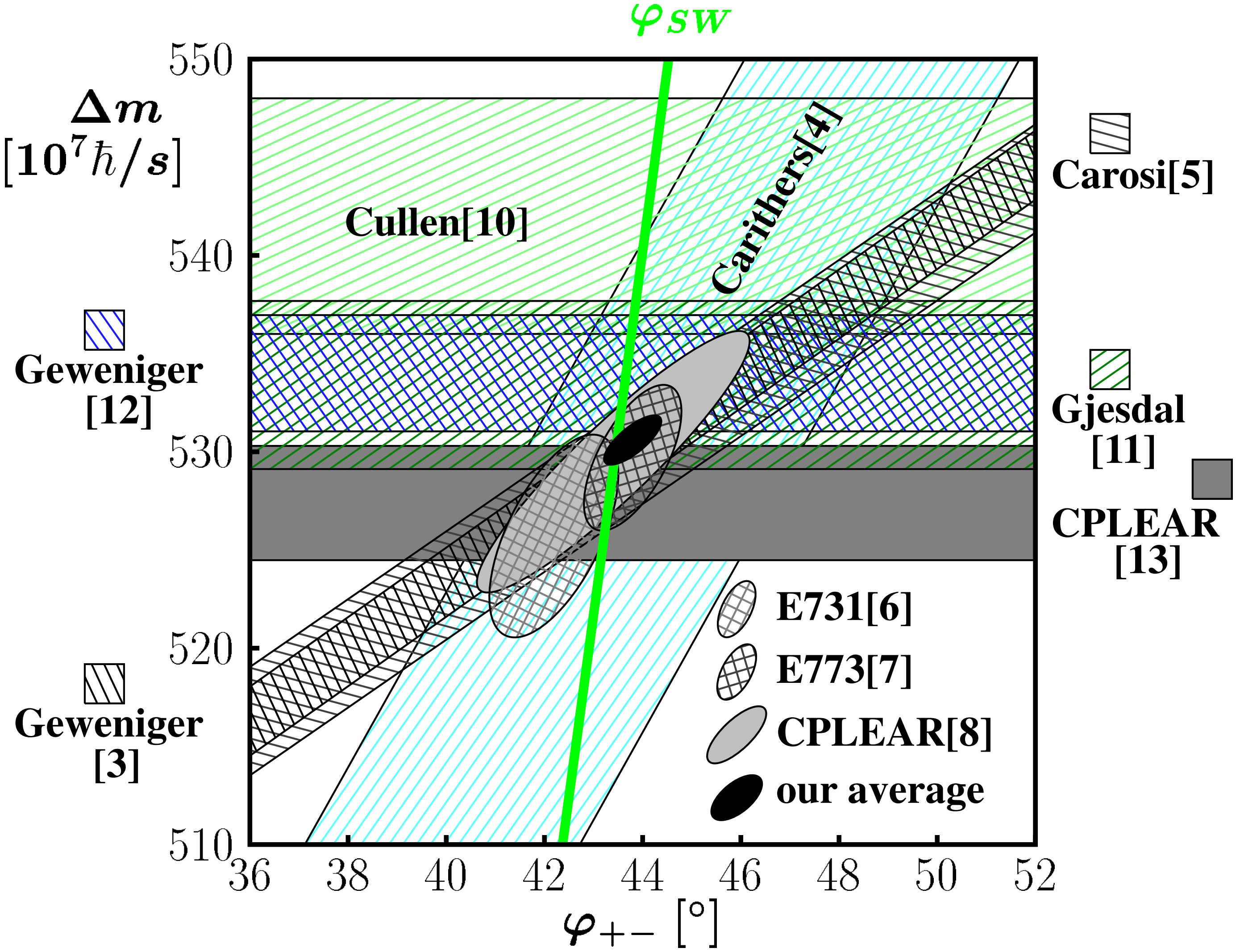

Figure 1: The plot shows the 1σ contour plots of all measurements listed in Table 1 and 2. The black ellipse in the center represents the result of our fit. The expected region for the value of φ_{SW} is also shown.